USN | | | | | | | | | | | **06MAT11**

First Semester B.E. Degree Examination, June 2012

Engineering Mathematics - I

Time: 3 hrs. Max. Marks:100

Note: ***1. Answer FIVE full questions choosing at least two from each part.***
2. Answer all objective type questions only on OMR sheet page 5 of the Answer Booklet.
3. Answers to objective type questions on sheets other than OMR will not be valued.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

PART - A

1 a. Choose the correct answer : **(04 Marks)**

i) The n^{th} derivative of $\log[(3x + 1)e^{9x+5}]$ is

(A) $\dfrac{3^n(-1)^{n-1}(n-1)!}{(3x+1)^n}$ (B) $\dfrac{3^n(-1)^n n!}{(3x+1)^{n+1}}$

(C) $\dfrac{n!(-1)^n}{(3x+1)^n}$ (D) Zero.

ii) If ϕ is the angle between radius vector and tangent to the curve $r = f(\theta)$, then $\tan\phi$ is

(A) $\dfrac{1}{r}\dfrac{d\theta}{dr}$ (B) $r\dfrac{d\theta}{dr}$ (C) $r\dfrac{dr}{d\theta}$ (D) $\dfrac{1}{r}\dfrac{dr}{d\theta}$

iii) If $r = e^{\theta}$ at $\theta = 0$, then the slope of the curve is

(A) 0 (B) $\pi/4$ (C) $\dfrac{\pi}{2}$ (D) 1

iv) The angle between the radius vector and tangent for the curve $r = \alpha\, e^{\theta \cot \alpha}$ is

(A) 1 (B) $\dfrac{\pi}{2}+\alpha$ (C) π (D) α

b. Find the nth derivative of $e^{-3x}\cos^2 6x$. **(04 Marks)**

c. If $y = \log(x+\sqrt{1+x^2})$, prove that $(1+x^2)y_{n+2} + (2n+1)xy_{n+1} + n^2y_n = 0$. **(06 Marks)**

d. Find the pedal equation of the curve $r^m = a^m(\cos m\theta + \sin m\theta)$. **(06 Marks)**

2 a. Choose the correct answer : **(04 Marks)**

i) If $u = x^2 + y^2$ then $\dfrac{\partial^2 u}{\partial x \partial y}$ is equal to

(A) 2 (B) 0 (C) 2x + 2y (D) x + y

ii) If $u = \log\left(\dfrac{x^2}{y}\right)$, then $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y}$ is equal to

(A) 2u (B) u (C) 0 (D) 1

iii) If $x = r\cos\theta$, $y = r\sin\theta$, then

(A) $\dfrac{\partial x}{\partial r} = \dfrac{1}{\partial r/\partial x}$ (B) $\dfrac{\partial x}{\partial r} = \dfrac{\partial r}{\partial x}$ (C) $\dfrac{\partial x}{\partial r} = 0$ (D) None of these

iv) If an error of 1% is made in measuring its length and breadth, the percentage error in the area of a rectangle is

(A) ·2% (B) 2% (C) ·02% (D) 1%

b. If $u = \tan^{-1}\left(\frac{x^3y^3}{x^3+y^3}\right)$ prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{3}{2}$ sin 2u. **(04 Marks)**

c. If z = f(x, y), $x = e^u + e^{-v}$ and $y = e^{-u} + e^v$, show that $\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y}$. **(06 Marks)**

d. If u = x + y + z , $v = x^2 + y^2 + z^2$, w = xy + yz + zx, find $J\left(\frac{u, v, w}{x, y, z}\right)$. **(06 Marks)**

3 a. Choose the correct answer : **(04 Marks)**

i) The value of $\int_{-\pi/2}^{\pi/2} \cos^8 x dx$ is

(A) $\frac{32\pi}{35}$ (B) $\frac{32}{35}$ (C) Zero (D) $\frac{35\pi}{128}$

ii) The value of $\int_0^\infty \frac{x^2}{(1+x^2)^{7/2}} dx$ is

(A) $\frac{4}{15}$ (B) $\frac{2}{15}$ (C) $\frac{2\pi}{15}$ (D) $\frac{15}{2}$

iii) If the equation of the curve remains unchanged after changing θ by –θ, the curve r = f(θ) is symmetrical about

(A) Initial line (B) the pole
(C) Symmetry does not exist (D) None of these.

iv) $\int_0^{\pi/4} \tan^5 \theta d\theta =$

(A) 2 log 2 (B) 2 log 2-1 (C) $\frac{1}{4}$(2log 2-1) (D) $\log\left(\frac{2}{e}\right)$

b. Obtain the reduction formula for $\int \sec^n x dx$. **(04 Marks)**

c. Evaluate $\int_0^{2a} x^2\sqrt{2ax - x^2}$ dx. **(06 Marks)**

d. Trace the curve $a^2y^2 = x^2(a^2 - x^2)$. **(06 Marks)**

4 a. Choose the correct answer : **(04 Marks)**

i) If r = f(θ) be the polar curve, then $\frac{ds}{dr}$ is

(A) $\sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}$ (B) $\sqrt{r^2 + \left(\frac{d\theta}{dr}\right)^2}$ (C) $\sqrt{1 + r^2\left(\frac{d\theta}{dr}\right)^2}$ (D) None of these

ii) The area bounded by the curve y = f(x), the x – axis and the ordinate x = a and x = b is

(A) $\int_a^b y dy$ (B) $\int_a^b y dx$ (C) $\int_b^a y dx$ (D) None of these

iii) The length of the curve $y = \frac{2}{3} x^{3/2}$ between x = 1 and x = 4 is

(A) $\frac{2}{5}\left(5^{3/2} + 2^{3/2}\right)$ (B) $\frac{2}{3}\left(5^{3/2} - 2^{3/2}\right)$ (C) $\frac{2}{3}5^{3/2}$ (D) $\frac{2}{3}2^{3/2}$

iv) $\frac{d}{d\alpha}\left[\int_a^b f(x,\alpha)dx\right]$ is equal to

(A) $\int_a^b \frac{d}{d\alpha} f(x, \alpha)dx$ (B) $\int_a^b \frac{d}{d\alpha} f(x, \alpha)dx$
(C) $\int_a^b \frac{\partial}{\partial x} f(x, \alpha)\, d\alpha$ (D) None of these.

b. If $x = a e^t \sin t$, $y = a e^t \cos t$, find $\frac{ds}{dt}$. **(04 Marks)**

c. Find the surface of the solid formed by revolving the cardiode $r = a(1+\cos\theta)$ about the initial line. **(06 Marks)**

d. Evaluate $\int_0^\infty \frac{e^{-x}}{x}(1-e^{-\alpha x})dx$, when $\alpha > -1$ by differentiating under the integral sign. **(06 Marks)**

PART - B

5 a. Choose the correct answer : **(04 Marks)**

i) The general solution of $xdy - ydx = 0$ is

(A) $x + y = c$ (B) $xy = c$ (C) $y = xc$ (D) None of these

ii) The integrating factor of the differential equation $x^2 \frac{dy}{dx} + y = 1$ is

(A) $\frac{1}{x^2}$ (B) $\log x$ (C) $e^{-1/x}$ (D) None of these

iii) The homogeneous differential equation $M(xy)\,dx + N(xy)\,dy = 0$ can be reduced to a differential equation in which the variables are separated by the substitution

(A) $x + y = v$ (B) $x = v/y$ (C) $y = vx$ (D) $y - x = v$

iv) The equation $y - 2x = c$ represents the orthogonal trajectories of the family

(A) $y = a e^{-2x}$ (B) $x + 2y = c$ (C) $xy = a$ (D) $x^2 + 2y^2 = a$

b. Solve $\cos(x+y+1)\,dx - dy = 0$. **(04 Marks)**

c. Solve $(1+ e^{x/y})\,dx + e^{x/y}(1 - x/y)\,dy = 0$. **(06 Marks)**

d. Show that the orthogonal trajectories of the family of cardiode $r = a\cos^2(\theta/2)$ is another family of cardiode $r = b\sin^2(\theta/2)$. **(06 Marks)**

6 a. Choose the correct answer : **(04 Marks)**

i) Let $\sum u_n$ be a series of +ve terms. Given that $\sum u_n$ is convergent and also $\lim_{n\to\infty} \frac{u_{n+1}}{u_n}$ exists then the said limit is

(A) Necessarily equal to one (B) Necessarily greater than one

(C) May be equal to one or less than one (D) Necessarily less than one.

ii) The series $\frac{2}{1^2} - \frac{3}{2^2} + \frac{4}{3^2} - \frac{5}{4^2} + ...$ is

(A) Conditionally convergent (B) Absolutely convergent

(C) Divergent (D) None of these.

iii) Which one of the following series is not convergent?

(A) $\frac{1}{2\sqrt{2}} + \frac{1}{3\sqrt{3}} + \frac{1}{4\sqrt{4}} + ...$ (B) $\frac{3}{2} - \frac{4}{3} + \frac{5}{4} - \frac{6}{5} + ...$

(C) $\frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5} + ...$ (D) $x + x^2 + x^3 + ...$ where $|x| < 1$.

iv) If $\sum u_n$ is +ve term infinite series and if $\lim_{n\to\infty} u_n = 0$, then $\sum u_n$ is

(A) Convergent (B) Divergent

(C) Either convergent or divergent (D) Oscillatory.

b. Find the nature of the series $\frac{2}{1.2.3} + \frac{4}{2.3.4} + \frac{6}{3.4.5} + ...$ **(04 Marks)**

c. Test for convergence of the series : $1 + \frac{3}{7}x + \frac{3.6}{7.10}x^2 + \frac{3.6.9}{7.4.5}x^3 + ... +$, $x > 0$. **(06 Marks)**

d. Define absolute convergence and conditional convergence. Is the series $1-\frac{1}{2\sqrt{2}}+\frac{1}{3\sqrt{3}}-\frac{1}{4\sqrt{4}}+\ldots$ conditionally convergent? **(06 Marks)**

7 a. Choose the correct answer : **(04 Marks)**

i) If $\cos\alpha$, $\cos\beta$, $\cos\gamma$ be the direction ratios of the line, then $\sin^2\alpha + \sin^2\beta + \sin^2\gamma =$

(A) 2 (B) 6 (C) 4 (D) 8

ii) The angle between the two planes $2x - y - 3z = 5$ and $x + 3y - 2z + 6 = 0$ is

(A) 60^0 (B) 90^0 (C) $\cos^{-1}(\frac{5}{14})$ (D) None of these

iii) The angle between any two diagonals of a cube is

(A) $\pi/3$ (B) $\tan^{-1}(1/3)$ (C) $\cot^{-1}(1/3)$ (D) $\cos^{-1}(1/3)$

iv) The equation of a straight line parallel to the x – axis is given by

(A) $\frac{x-a}{1}=\frac{y-b}{1}=\frac{z-c}{1}$ (B) $\frac{x-a}{0}=\frac{y-b}{1}=\frac{z-c}{1}$

(C) $\frac{x-1}{1}=\frac{y-b}{0}=\frac{z-c}{0}$ (D) $\frac{x-a}{0}=\frac{y-b}{0}=\frac{z-c}{1}$

b. Find the value of k, such that the set of four points (1,1,0), (1,2,1), (4,5,6) and (3,0,k) are coplanar. **(04 Marks)**

c. Find the equation of the plane passing through the line of intersection of the plane $2x + y - z = 1$, $5x - 3y + 4z + 3 = 0$ and parallel to the line $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$. **(06 Marks)**

d. Find the shortest distance and its equation between the lines $\frac{x-6}{3}=\frac{y-7}{-1}=\frac{z-4}{1}$ and $\frac{x}{-3}=\frac{y-9}{2}=\frac{z-2}{4}$. **(06 Marks)**

8 a. Choose the correct answer : **(04 Marks)**

i) The angle between the two surfaces $\phi(x,y,z)$ and $\psi(x,y,z)$ at any point (x_1,y_1,z_1) is $\theta =$

(A) $\text{Sin}^{-1}\left(\frac{\nabla\phi\cdot\nabla\Psi}{|\nabla\phi||\nabla\psi|}\right)$ (B) $\text{Cos}^{-1}\left(\frac{\nabla\phi\cdot\nabla\Psi}{|\nabla\phi||\nabla\psi|}\right)$

(C) $\tan^{-1}\left(\frac{\nabla\phi\cdot\nabla\Psi}{|\nabla\phi||\nabla\psi|}\right)$ (D) None of these

ii) A unit tangent vector to the surface $x = t$, $y = t^2$, $z = t^3$ at $t = 1$.

(A) $\left(1/2, 1/3, 1/4\right)$ (B) $\left(\frac{1}{\sqrt{14}},\frac{2}{\sqrt{14}},\frac{3}{\sqrt{14}}\right)$

(C) $\left(\sqrt{2},\sqrt{3},\sqrt{4}\right)$ (D) None of these

iii) If $A = 2x^2 i - 3yzj + xz^2 k$, then $\nabla.A$ is

(A) $4x - 3z + 2xz$ (B) $xi + yj + zk$

(C) $2(xi + 4yj + 3zk)$ (D) None of these.

iv) For any scalar $\phi(x,y,z)$, the value of $\nabla x \nabla\phi$ is

(A) 1 (B) 0 (C) 2 (D) None of these

b. Find the directional derivative of $\phi = x^2yz + 4xz^2$ at the point (-1, -2, -1) in the direction of the vector $2i - j - 2k$. **(04 Marks)**

c. If $\vec{A}$ is a vector function and ϕ is a scalar function then prove that $\text{Curl}(\phi\,.\,\vec{A}) = \phi(\text{curl}\,\vec{A}) + \text{grad}\,\phi \times \vec{A}$. **(06 Marks)**

d. Find the constants a and b, so that $\vec{F} = (axy + z^3)i + (3x^2 - z)j + (bxz^2 - y)k$ is irrotational and find ϕ such that $\vec{F} = \nabla\phi$. **(06 Marks)**

*****